Being Precise about the Uncertainty Associated with Orbital Parameters

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Real Trajectories

- Real evolution:

$$\dot{x}_t = f(x_t) + w_t \tag{1}$$

  - x_t are the orbital parameters at (continuous) time t;
  - $f(x_t)$ is the real (e.g., non-Keplerian) force model;
  - w_t is the unknown input (e.g., due to manoeuvres) at time t;

Precise Modelling of Trajectories

- Model for evolution:

$$\dot{x}_t = f(x_t) + w_t + \left(\hat{f}(x_t) - \hat{f}(x_t)\right) \quad (2)$$

$$= \hat{f}(x_t) + w_t + \left(f(x_t) - \hat{f}(x_t)\right) \quad (3)$$

$$\approx \hat{f}(x_t) + \hat{w}_t \quad (4)$$

  - $\hat{f}(x_t)$ is the precise force model;
  - $\hat{w}_t = w_t + \left(\hat{f}(x_t) - f(x_t)\right)$ includes the mismodelling error.

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  - $\hat{f}(x_t)$ is the precise force model;
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- Uncertainty stems from unknown x_0 and non-zero $\hat{w}_t$:
  - Errors in x_0 would have an effect that decays with time;
  - Errors in $f(.)$ have an effect that accumulates with time;
  - Numerical errors have an effect that accumulates with time.

Precise Measuring of Trajectories

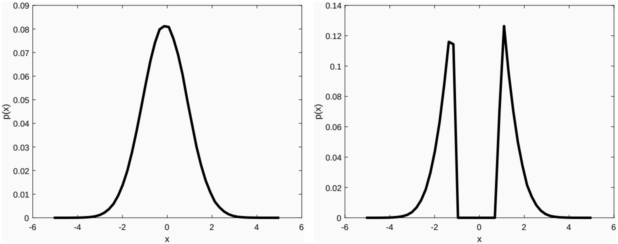
- At iteration k, we get measurements, y_k;
- Measurements typically provide information about x_k:

$$y_k = h(x_k) + e_k \tag{5}$$

  - $h(x_k)$ is the projection of the orbital parameters into the sensor's field of view;
  - e_k is the measurement error.
- Uncertainty stems from:
  - Reducing the size of e_k still means it is non-zero;
  - $h(x_k)$ is very unlikely to measure x_k directly;

Being Precise About Uncertainty

- Precise modelling and measuring still results in uncertainty
- Uncertainty can be non-Gaussian (i.e., not a TLE)
  - E.g. due to negative information or nonlinear $f(.)$ and $h(.)$



(a) Gaussian

(b) Not a Gaussian

Figure: An uncertain orbital parameter

Being Precise About Uncertainty

- We advocate using 'samples' to approximate the uncertainty

$$p(x) \approx \hat{p}(x) = \sum_{i=1}^{N} w^{(i)} \delta\left(x - x^{(i)}\right) \tag{6}$$

  - $x^{(i)}$ is a hypothesised value for x;
  - $w^{(i)}$ is an associated weight;
  - N is the number of samples.

- We don't have to approximate our precise models and yet:

$$\lim_{N\to\infty} \left[\sigma^2\right] = \lim_{N\to\infty} \left[\frac{c}{N}\right] = 0 \tag{7}$$

  - c is a constant for any problem and (valid) sampling scheme;
  - σ^2 is the error resulting using $\hat{p}(x)$ not $p(x)$

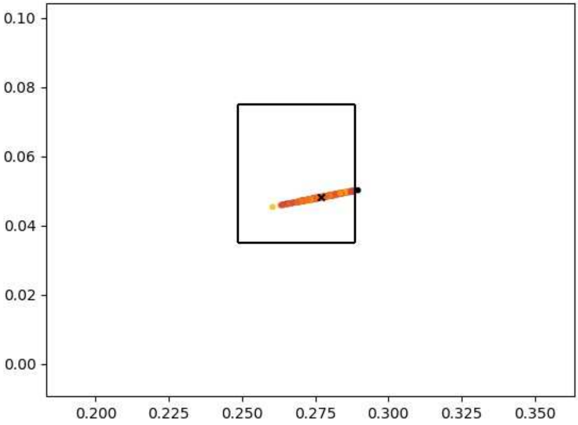
Being Precise About Uncertainty

- This sampling paradigm can handle a range of problems:
  - Estimating a fixed parameter from fixed dataset (e.g., MCMC);
  - Estimating a fixed parameter from an ever-growing dataset (e.g., Fixed Lag-Sequential Monte Carlo Samplers, FL-SMC);
  - Estimating a time-evolving state from an ever-growing dataset (e.g., Particle Filters).
- How you select $x^{(i)}$ matters (for fixed N)
  - Computationally efficient accurate schemes (e.g., NUTS) exist.
- The alternative is to approximate the precise trajectories and measurements
  - This leads to Gaussian assumption (implicit in least-squares);
  - Can be extended to Gaussian mixture but you still approximate.

Maintaining Track Custody Using Negative Information

- FL-SMC (using NUTS) applied to simulated telescope data;
  - Hourly tasking adaptively maximising probability of detection;
  - Plan is to apply this to using the Liverpool Telescope
    - Hope to test with MEV-2 in GTO during October 2020.
- Graph's axes are right ascension and declination;
- Telescope field of view shown as a black box;
- Dots represent $h\left(x_k^{(i)}\right)$, where we think objects are in the sky;
- Colours represent $w_k^{(i)}$

Maintaining Track Custody Using Negative Information



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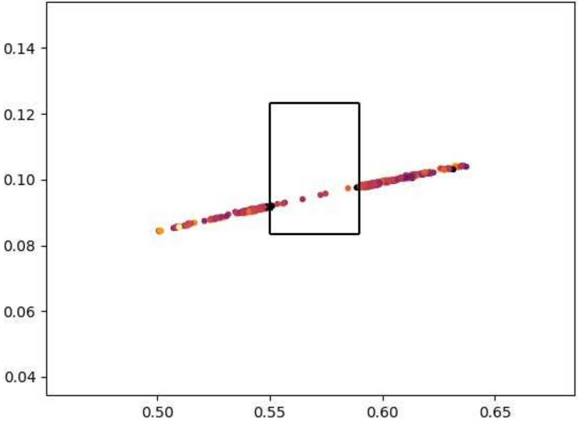

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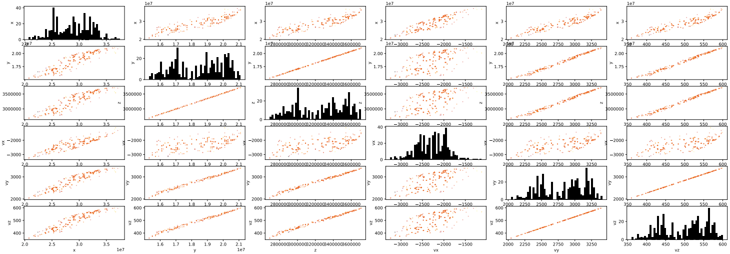

Figure: Resulting uncertainty in (3D) Cartesian position and velocity

Maintaining Track Custody Using Negative Information

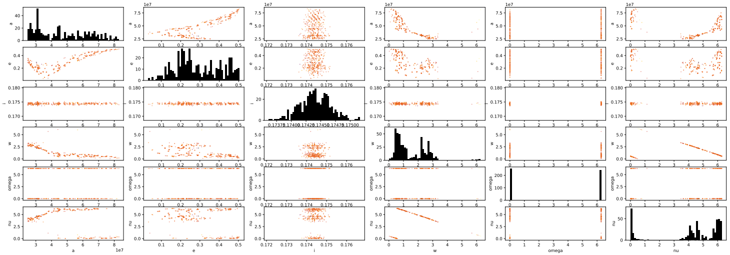

Figure: Resulting uncertainty in (Keplarian) orbital parameters

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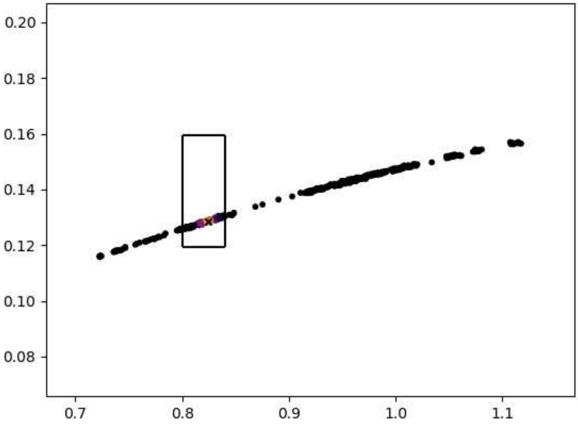

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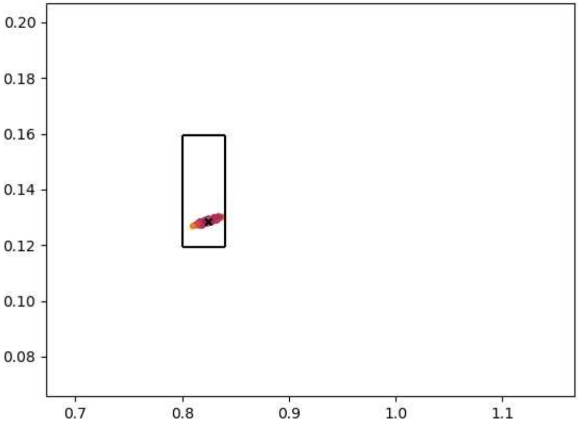

Conclusions

- Uncertainty (still) stems from precise trajectory models and measurements;
- Sample-based approaches can be precise about uncertainty and facilitate:
  - Initial orbit determination (including searching for objects);
    - Accumulation of information over long time-scales (e.g., for detecting small bits of debris);
  - (Retrospective) manoeuvre detection;
  - Improved conjunction analysis;
  - Interfacing to Stone Soup to:
    - Use mature algorithms for data association (e.g., MHT);
    - Fuse data from each of multiple sensors.