Uncertain parameters in Orbit Determination and their impact in SSA

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gmv
INNOVATING SOLUTIONS

OUTLINE

1. Uncertainty and covariance realism

- Custody maintenance
- Manoeuvre detection
- Rendezvous and close proximity operations
- Conjunction risk assessment

2. Covariance in SSA products

3. Uncertain parameters in BLS

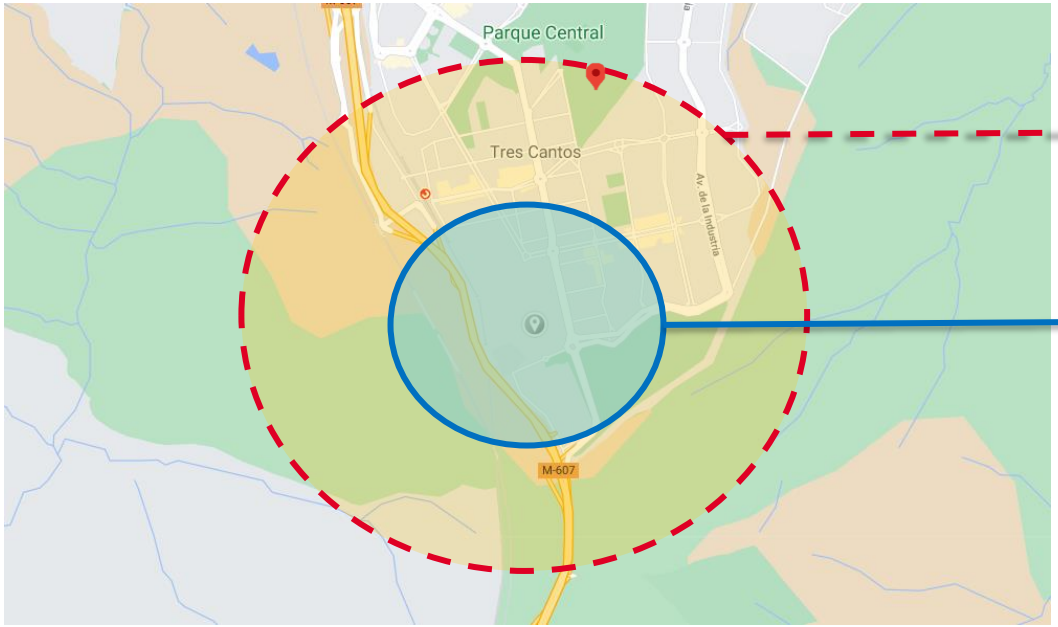
4. CONCLUSIONS

Uncertainty realism

Uncertainty realism ⇨ How well characterized the uncertainty is

What happens if the **solid circle** is not realistic, and in fact we are outside of it?

completely lost!

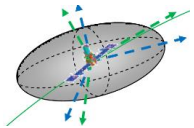


Realistic uncertainty

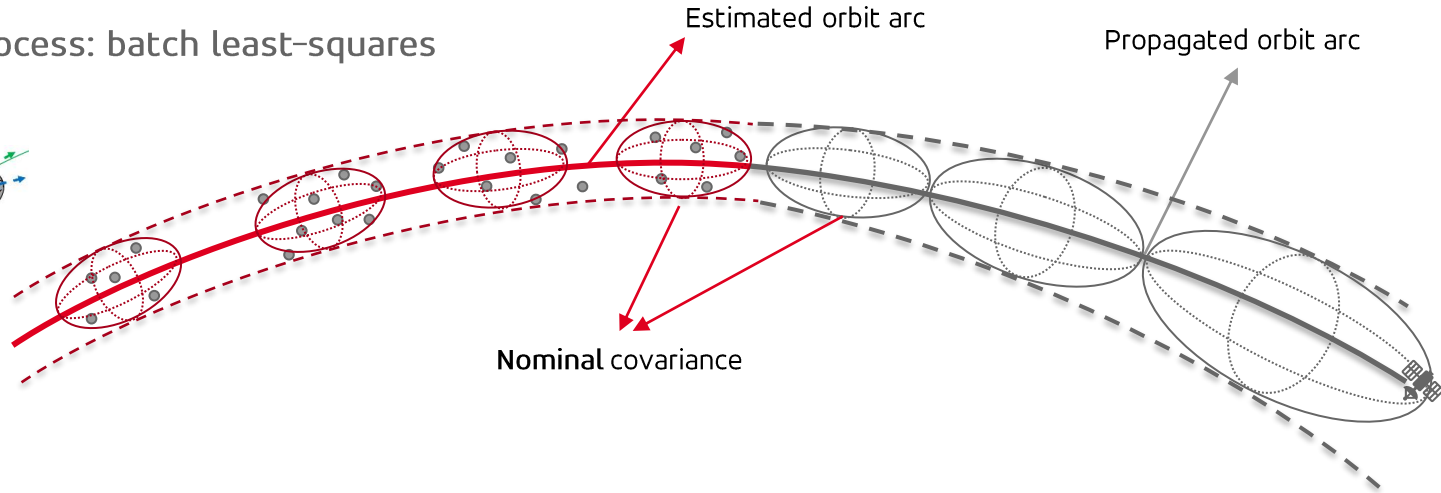
Non-realistic uncertainty

Uncertainty Realism in Orbit Determination and Propagation

- Orbit determination process: batch least-squares
  - State
  - Covariance



Uncertainty representation under Gaussian assumptions



$$P^n = \left(H^T W H\right)^{-1}$$

Observations weighting matrix (expected noise)

Jacobian of the observations with respect to the estimated parameters

Uncertainty Realism in Orbit Determination and Propagation

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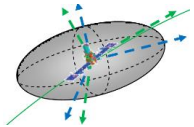
  - State
  - Covariance



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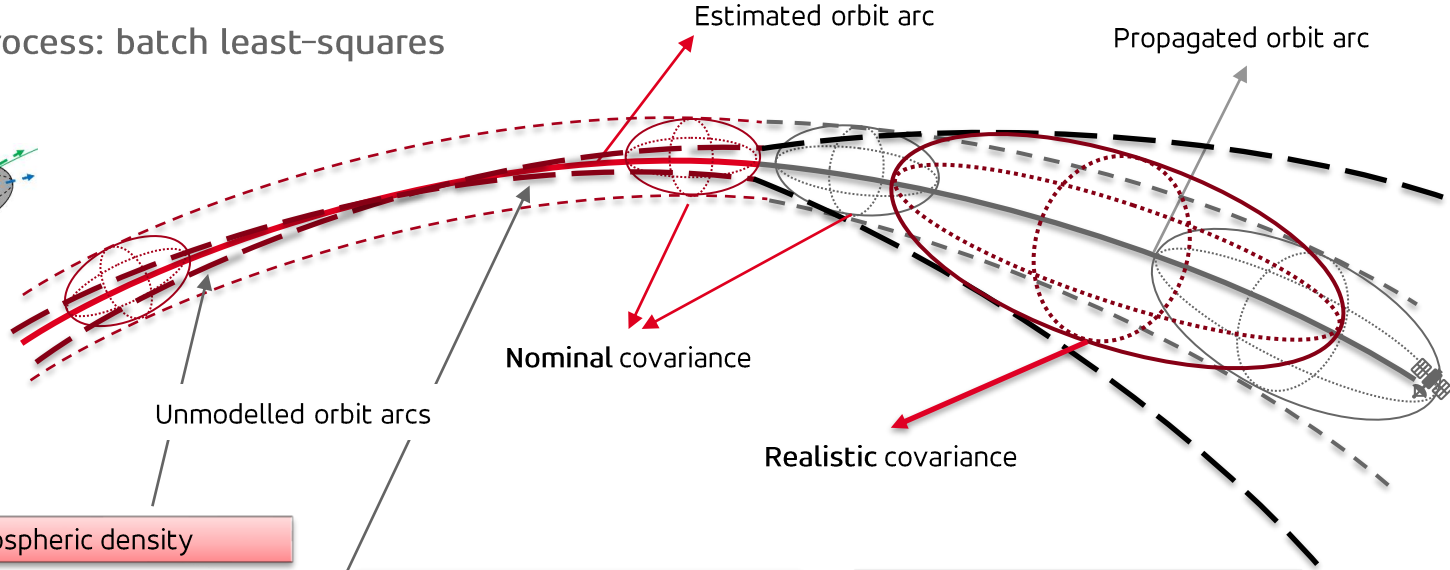


- Atmospheric density
- Range bias
- Solar proxies prediction
- Solar radiation pressure
- Time bias

Dynamical and measurements models are not perfect!

Lack of covariance realism

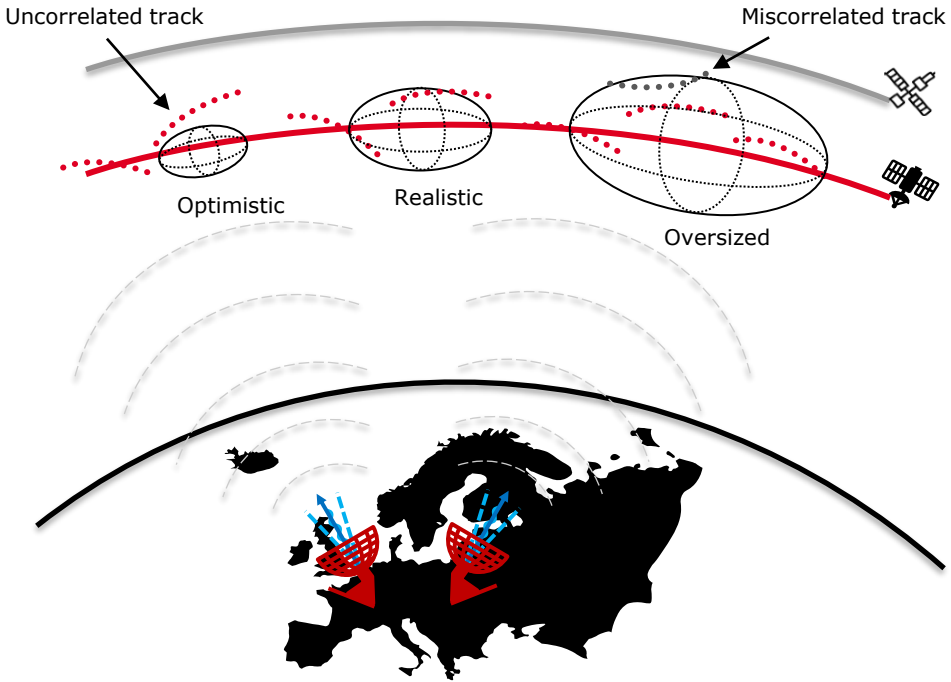
Problem to be ~~solved~~ **mitigated**

Nominal OD processes do not account for the **model uncertainty**

Covariance impact on SSA products

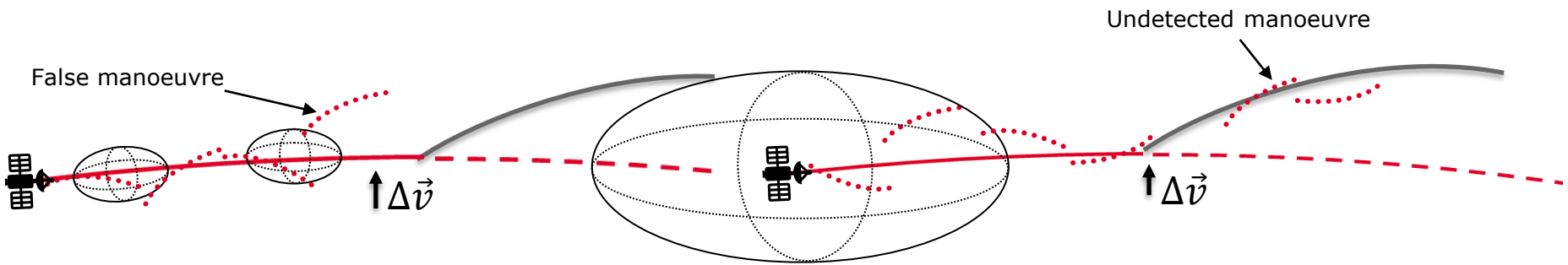
Custody maintenance

- RSO catalog build-up and maintenance activities
- Covariance-based track association (CBTA) algorithms
  - Over-optimistic covariance → uncorrelated tracks (**false negatives**)
  - Oversized covariance → miscorrelated tracks, **cross-tagging**



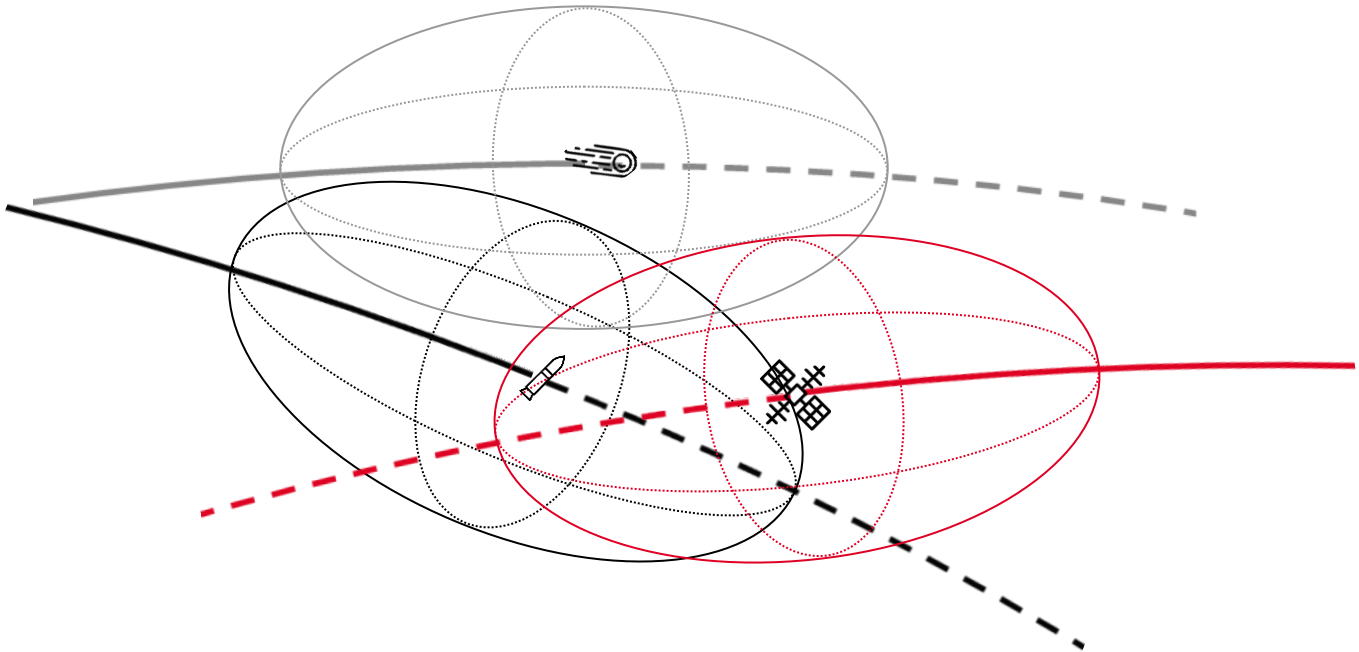
Manoeuvre detection

- **Undersized covariance**
  - Rise of false manoeuvre detections
  - Very problematic with increasing number of objects
- **Oversized covariance**
  - Lower sensitivity of detection algorithms
  - Problematic for small manoeuvres or electric propulsion



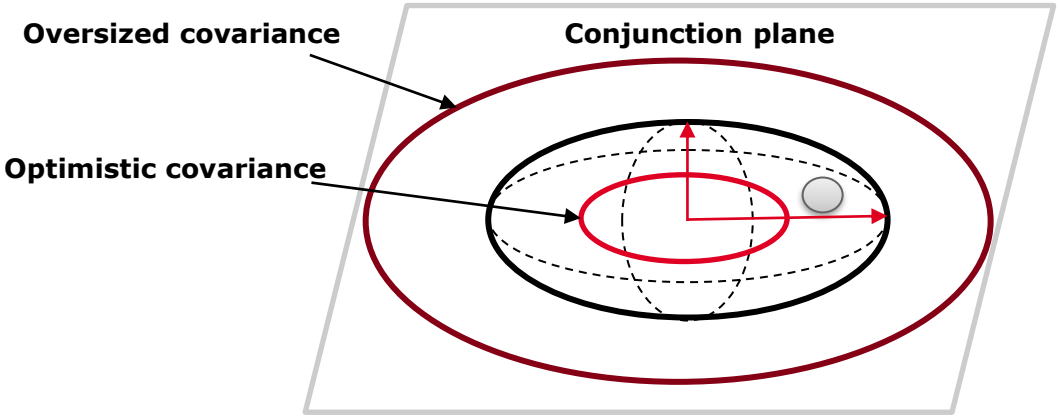
Rendezvous and close Proximity Operations (RPOs)

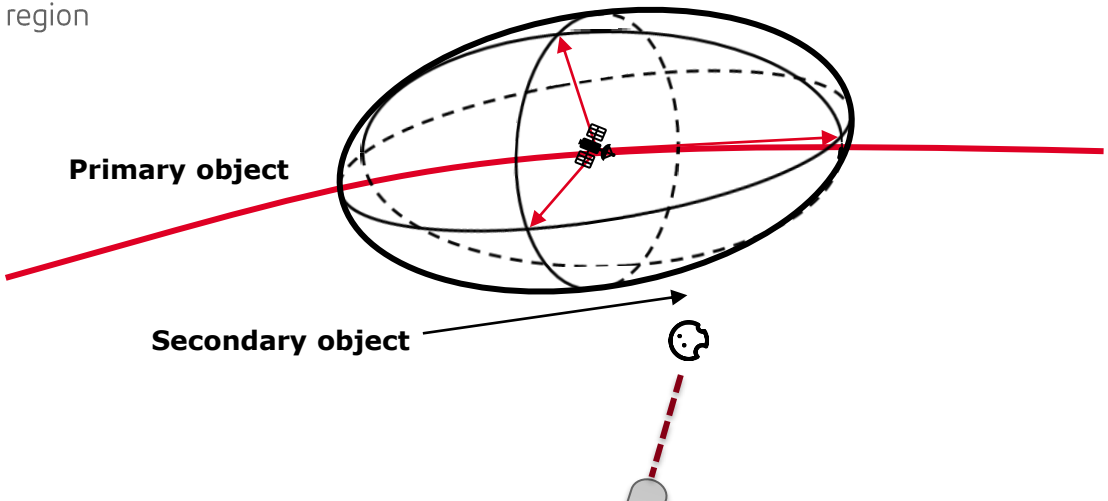
- Debris removal, in-orbit services, etc..
  - Trajectory design algorithms very sensitive to uncertainty
  - Dependent on custody maintenance: close-by flights or debris clouds



Collision risk analysis

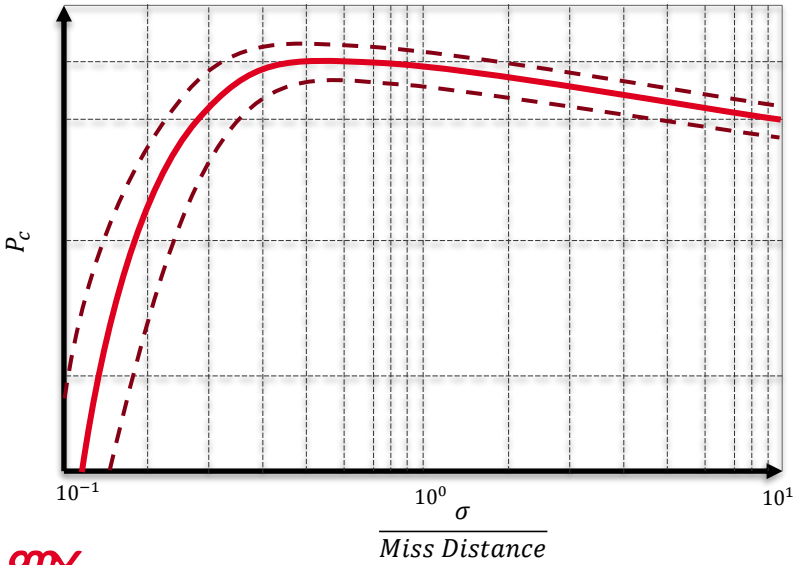
- Short-duration encounters
  - Gaussian assumption
  - Velocity uncertainty neglected
  - Spherical shape, combined radius
  - Rectilinear motion
  - PoC: Akella-Alfriend method
    - Integral of the covariance over the conjunction region

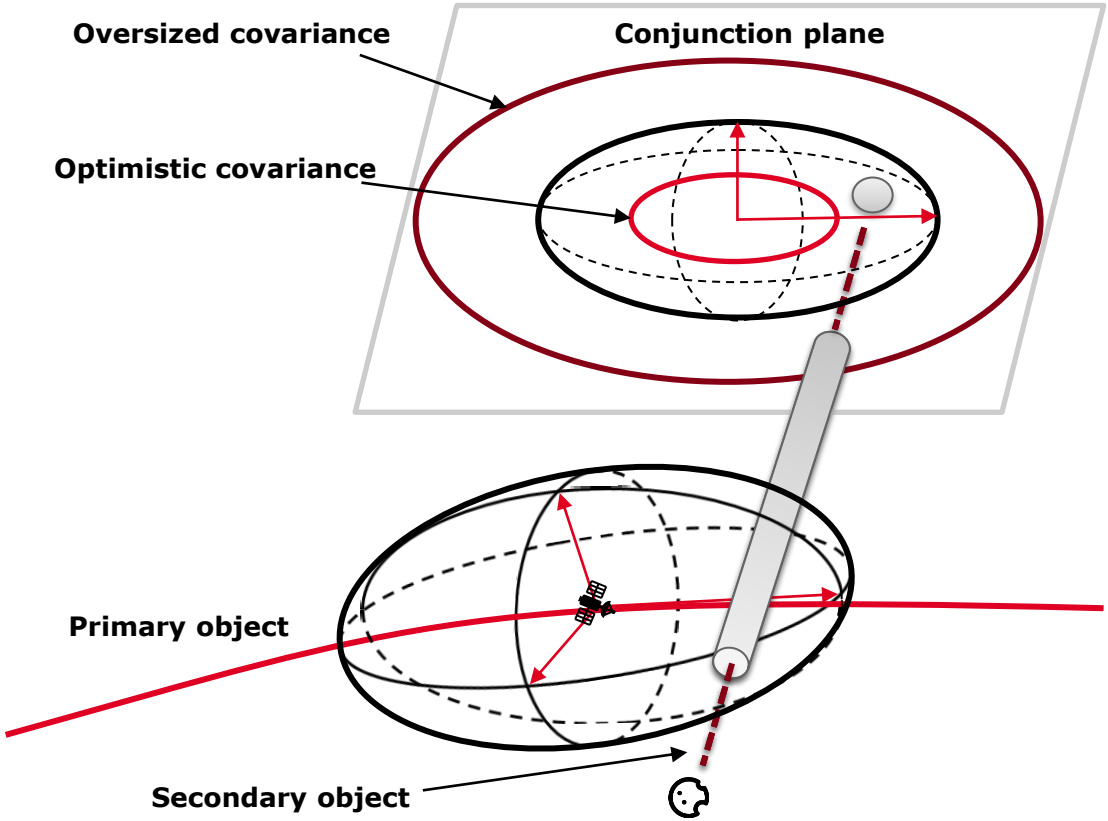




Collision risk analysis

- Covariance size impact
  - Depends on the conjunction geometry
  - Overly optimistic covariances → No warning
  - Oversized covariances → Under-estimation of risk





Most relevant uncertain parameters in batch least-squares Orbit Determination

Uncertain parameters

- Atmospheric drag
- Solar proxies prediction
- Measurements bias
- Solar radiation pressure
- Time bias

$$\mathbf{a}_{\text{drag}} = -\frac{1}{2}\rho\frac{C_d A}{m}|\mathbf{v}_{\text{rel}}|^2\frac{\mathbf{v}_{\text{rel}}}{|\mathbf{v}_{\text{rel}}|}$$

$$\mathbf{a}_{\text{drag}} = -\frac{1}{2}\rho\frac{C_d A}{m}|\mathbf{v}_{\text{rel}}|^2\frac{\mathbf{v}_{\text{rel}}}{|\mathbf{v}_{\text{rel}}|}\left(1 + c_{PE}\cdot t_{\text{pred}}\right)$$

$$z^* = z + c_z$$

$$\mathbf{a}_{SRP} = -P_{SRP}C_R\frac{A}{m}\frac{r}{r^3}AU^2$$

$$t^* = t + c_{TB}$$

A. Cano et al., "Covariance determination for improving uncertainty realism in orbit determination and propagation" https://doi.org/10.1016/j.asr.2022.08.001

A. Cano et al., "Catalog-Based Atmosphere Uncertainty Quantification" https://doi.org/10.1007/s40295-023-00403-w

A. Cano et al., "Covariance propagation with time-correlated errors" 2nd NEO and Debris Detection Conference

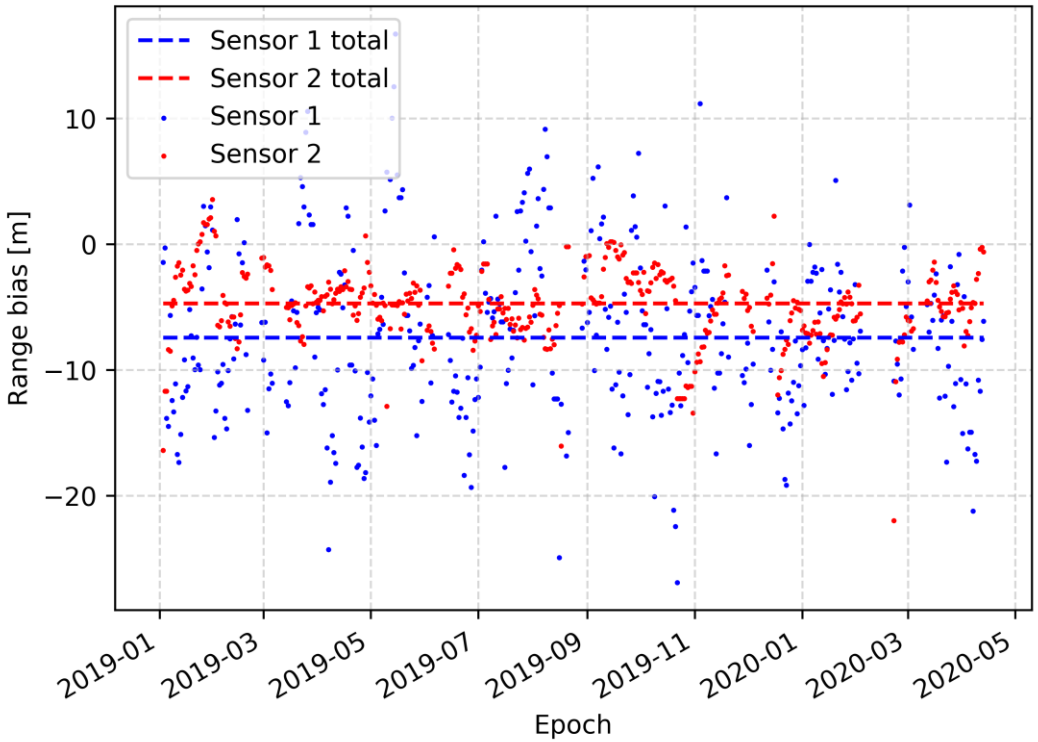
A. Cano et al., "Stochastic Consider Parameters for covariance realism in orbit determination" 2023 AAS/AIAA Astrodynamics Specialist Conference

A. Cano et al., "Improving orbital uncertainty realism through covariance determination in GEO" https://doi.org/10.1007/s40295-022-00343-x

Measurements uncertain biases

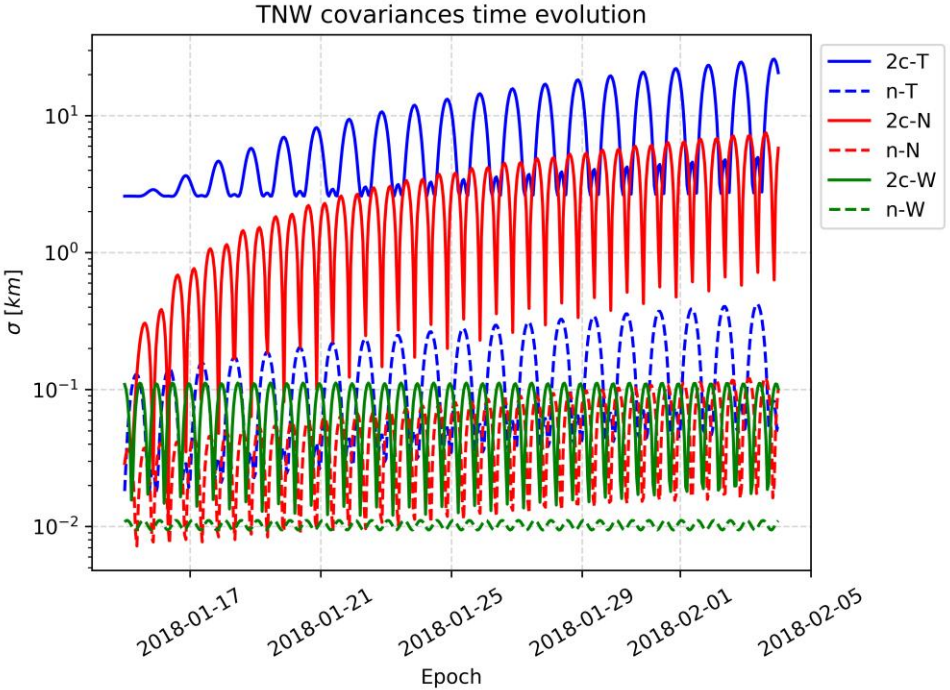
- The sensor bias depends on many factors
  - Sensor type: range bias, range-rate bias, clock time bias...
  - Sensor technology, location, astrometry algorithm, etc.
  - Sensor performances vary depending on the orbital regime and orbit geometry
- The sensor biases are not constant
  - They are averaged during certain calibration periods
  - They are updated seldomly

$$z^* = z + c_z$$



Dynamic and measurements models uncertainty impact

- Analysis of a GEO object
  - Uncertainty present in the SRP coefficient and the time bias of the telescopes
  - Comparison of a realistic versus an optimistic covariance (continuous vs dashed lines)
  - Measurements model uncertainty impacts the estimated covariance at estimation epoch
  - Dynamic model uncertainty is mapped into position with the propagation time



A. Cano et al., "Improving orbital uncertainty realism through covariance determination in GEO" https://doi.org/10.1007/s40295-022-00343-x

Atmospheric density variability

Objective

Estimate the uncertainty in density caused by the dispersion of F10.7 and Ap

Method

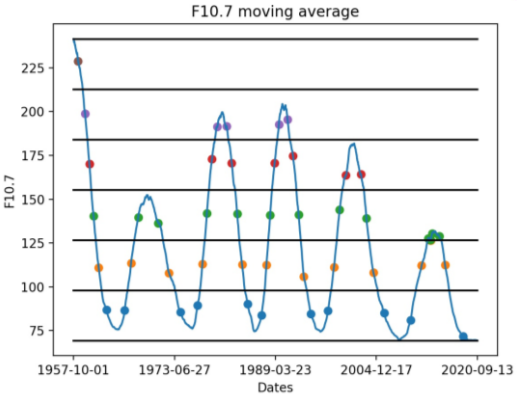
Statistical analysis of historic space weather data (Celestrak)

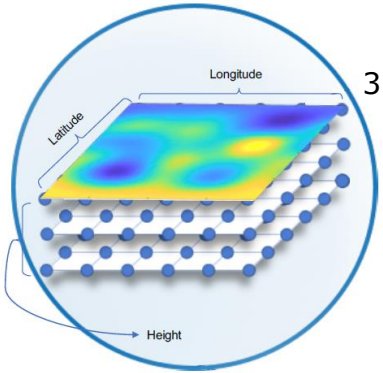
Results

Density uncertainty as a function of altitude and solar activity

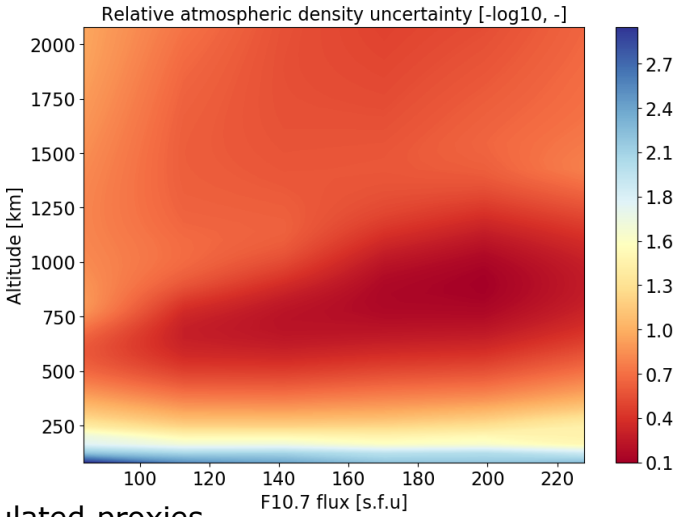
1. Choose solar activity strata and reference dates

2. Generate a 3D grid of lat/lon/alt



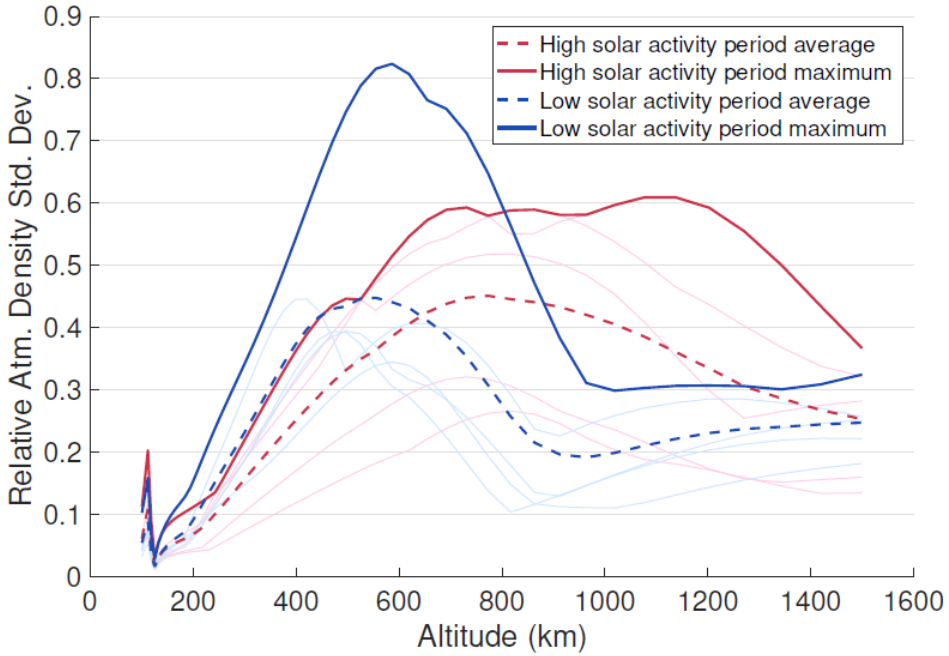


3. For each date:
   I. Gaussian distribution for the proxies based on the surrounding 60 days



4. For each grid point and epoch:
   I. Use NRLMSISE00 model using the simulated proxies
   II. Aggregate density outputs and compute their σ

Atmospheric density uncertainty evolution with altitude



Aguado et al., "Uncertainty propagation meeting space debris needs"
8th European Conference on Space Debris (2021)

Atmospheric density error stochastic modelling

NRLMSISE00

$$\rho(t) = \bar{\rho}(t) + z(t)$$

$$z(t_n) = a(n)z(t_{n-1}) + u(n)$$

$$u(n) \sim N\left(0, \sigma_u^2(n)\right)$$

$$\sigma_u^2(n) = r_0(t_n)\left[1 - \frac{r_0(t_n)}{r_0(t_{n-1})} e^{-2\lambda(t_n - t_{n-1})}\right]$$

$$a(n) = \frac{r_0(t_n)}{r_0(t_{n-1})} e^{-\lambda(t_n - t_{n-1})}$$

$\sigma_{atm}^2(t_n)$ Atmospheric density uncertainty (Previous slides)

$\lambda = \frac{1}{t_\lambda}$ Inverse of time scale

Parameter of interest

Limiting cases

Gaussian:
- no correlation

$t_\lambda \quad \lambda \longrightarrow \infty$

$z(t_n) = u(n)$

Constant noise:
- too strong correlation

$t_\lambda \quad \lambda \longrightarrow 0$

$z(t_n) = z(t_{n-1})$

A. Cano et al., "Covariance propagation with time-correlated errors"
2nd NEO and Debris Detection Conference

A. Cano et al., "Stochastic Consider Parameters for covariance realism in orbit determination"
2023 AAS/AIAA Astrodynamics Specialist Conference

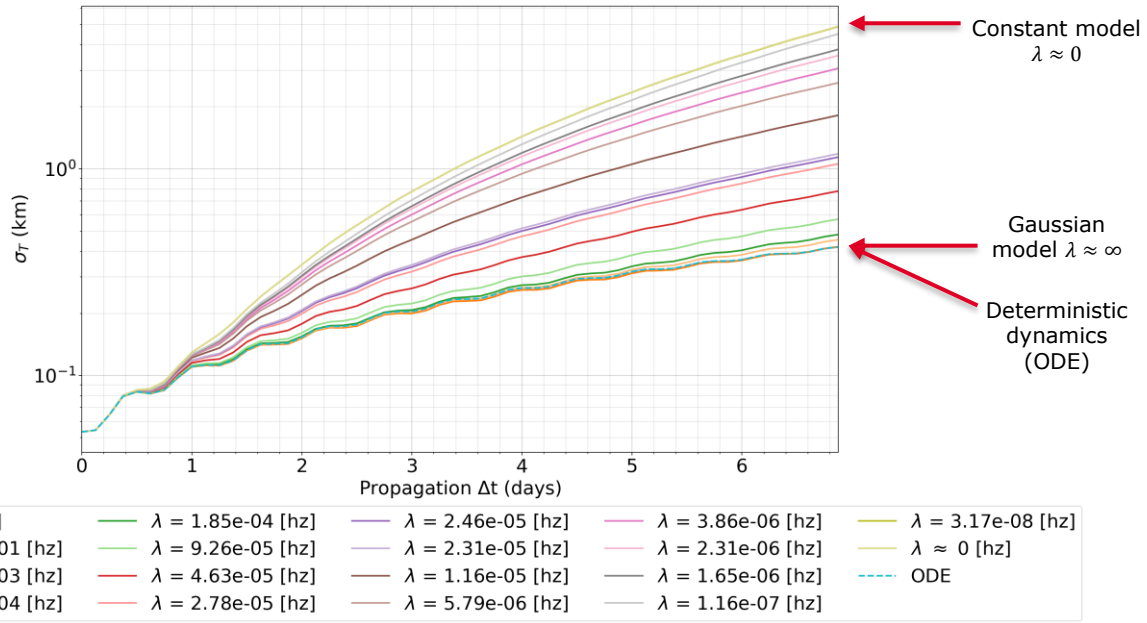
Impact of density error correlation in the covariance

800 km altitude circular LEO

Constant model $\lambda \approx 0$ ⟺ Greatest covariance

Smallest covariance

Deterministic dynamics (ODE) ⟺ Gaussian model $\lambda \approx \infty$



A. Cano et al., "Atmospheric correlation impact in uncertainty propagation"
3rd International Conference on Space Situational Awareness (IAA-ICSSA-22-0007)

Conclusions

01

SSA products depend on covariance realism

- Custody maintenance, manoeuvre detection
- RPOs, collision risk assessment

02

Uncertain parameters models

- Try to represent the errors present in the dynamic and measurement models
- Require a trade-off between accurate model and traceability and complexity

03

Drag force acceleration uncertainty

- Most difficult to model due to the multiple sources
- Stochastic models with correlation can be a good representation. But how to estimate it efficiently?

04

Uncertainty quantification techniques

- Are required to estimate the modelled uncertainty and improve the realism and reliability of SSA products

Thank you

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